Example 2. *By Gauss's elimination method, solve*

$$5x - y - 2z = 142$$
$$x - 3y - z = -30$$
$$2x - y - 3z = -50.$$

Solution. The largest coefficient in the first equation is 5, which is pivot element. So divide first equation by 5, we get

$$x - \frac{1}{5}y - \frac{2}{5}z = \frac{142}{5} \qquad ...(1)$$

Now eliminating x from second and third equations which help of (1), we get

$$-\frac{14}{5}y - \frac{3}{5}z = -\frac{292}{5} \qquad ...(2)$$

$$-\frac{3}{5}y - \frac{11}{5}z = -\frac{309}{5}. \qquad ...(3)$$

Eliminating y from (2) and (3), we get

$$-\frac{145}{5}z = -\frac{3450}{5}$$

$$z = \frac{3450}{145} = 23{\cdot}79.$$

Subtitute the value of z into (3), we get

$$-\frac{3}{5}y - \frac{11}{5}(23{\cdot}70) = -\frac{309}{5}$$

$$-\frac{3}{5}y = -\frac{309}{5} + \frac{11\,(23{\cdot}79)}{5}$$

$$-3y = -309 + 11\,(23{\cdot}79) = -309 + 261{\cdot}69$$

$$-3y = -47{\cdot}31$$

or $$y = 15{\cdot}77.$$

Substitute the values of y and z into (1), we get

$$x - \frac{1}{5}(15{\cdot}77) - \frac{2}{5}(23{\cdot}79) = \frac{142}{5}$$

$$x = \frac{142}{5} + \frac{15{\cdot}77}{5} + \frac{2\,(23{\cdot}79)}{5} = \frac{205{\cdot}35}{5}$$

or $$x = 41{\cdot}07.$$

Hence, the solution are given by

$$x = 41{\cdot}07,\ y = 15{\cdot}77,\ z = 23{\cdot}79.$$

Example 3. *Using Gauss's elimination method solve*

$$2x_1 + 4x_2 + x_3 = 3$$
$$3x_1 + 2x_2 - 2x_3 = 2$$
$$x_1 - x_2 + x_3 = 6.$$

Solution. Dividing first equation by 2, we get

$$x_1 + 2x_2 + \frac{1}{2}x_3 = \frac{3}{2}. \quad \text{...(1)}$$

Multiplying (1) by 3 and subtract from second and also subtract (1) from third equation, we get

$$4x_2 + \frac{7}{2}x_3 = \frac{5}{2} \quad \text{...(2)}$$

$$-3x_2 + \frac{1}{2}x_3 = \frac{9}{2}. \quad \text{...(3)}$$

Now dividing (2) by 4 and subtract after multiplied by – 3 from (3), we get

$$25x_3 = 51$$

or $$x_3 = \frac{51}{25} = 2{\cdot}04.$$

Substitute the value of x_3 in (2), we get

$$4x_2 + \frac{7}{2}(2{\cdot}04) = \frac{5}{2}$$

$$4x_2 = \frac{5}{2} - \frac{7\,(2{\cdot}04)}{2} = \frac{5 - 14{\cdot}28}{2}$$

$$x_2 = -\frac{9{\cdot}28}{8}$$

$$x_2 = 1{\cdot}16.$$

Now substitute the values of x_2 and x_3 in (1), we get

$$x_1 + 2(-1.16) + \frac{1}{2}(2.04) = \frac{3}{2}$$

$$x_1 = \frac{3}{2} + 2(1.16) - \frac{1}{2}(2.04)$$

$$= \frac{3 + 4.64 - 2.04}{2} = \frac{5.6}{2}$$

or $$x_1 = 2.8.$$

Hence, the solutions are given by

$$x_1 = 2.8, x_2 = -1.16, x_3 = 2.04.$$

Example 4. *Solve by Gauss-elimination mehtod*

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 23$$
$$4x + 11y - z = 33.$$

Solution. Dividing first equation by 2, we get

$$x + \frac{1}{2}y + 2z = 6. \quad ...(1)$$

Now subtract (1) after multiplied by 8 and 4 respectively from second and third equations, we get

$$-7y - 14x = -45 \quad ...(2)$$
$$9y - 9z = 9. \quad ...(3)$$

Now divide (2) by -7, we get

$$y + 2z = \frac{45}{7}. \quad ...(4)$$

Now multiplying (4) by 9 and subtract from (3), we get

$$-27z = 9 - \frac{405}{7}$$

or $$-27z = -\frac{342}{7}. \quad ...(5)$$

Hence, the system of equations reduces to upper triangular form as follows :

$$\left.\begin{aligned} x + \frac{1}{2}y + 2z &= 6 \\ y + 2z &= \frac{45}{7} \\ -27z &= -\frac{342}{7} \end{aligned}\right\}. \quad ...(6)$$

By back substitution, we get

$$z = \frac{342}{189} = 1.81$$

and $$y + 2(1.81) = \frac{45}{7}$$

$$y = \frac{45}{7} - 2(1.81)$$

$$= 6.43 - 3.62 = 2.81$$

and $$x + \frac{1}{2}(2.81) + 2(1.81) = 6.$$

$$\therefore \qquad x = 6 - \frac{1}{2}(2.81) - 2(1.81) = 0.975.$$

Hence, the solution is

$$x = 0.975,\ y = 2.81,\ z = 1.81.$$

Example 5. *Apply Gauss elimination method to solve the equations*

$$x + 4y - z = -5$$
$$x + y - 6z = -12$$
$$-3x - y - z = 4.$$

Solution. Eliminating x from second and third equation with the help of first equation. Subtract first equation from second and after multiplied by 3 from third respectively, we get the system of equations as follows :

$$x + 4y - z = -5 \qquad ...(1)$$
$$-3y - 5z = -7 \qquad ...(2)$$
$$-13y + 2z = 19. \qquad ...(3)$$

Eliminating y from (3) with help of (2). Divide (2) by -3 and then this equation is subtracted after multiplied by -13 from (3), we get

$$\left.\begin{aligned} x + 4y - z &= -5 \\ y + \frac{5}{3}z &= \frac{7}{3} \\ \frac{71}{3}z &= \frac{148}{3} \end{aligned}\right\}. \qquad ...(4)$$

By back substitution from (4), we get

$$z = \frac{148}{71}$$

and

$$y = \frac{7}{3} - \frac{5}{3}z$$
$$y = \frac{7}{3} - \frac{5}{3}\left(\frac{148}{71}\right)$$
$$y = -\frac{81}{71}$$

and

$$z = -5 - 4y + z$$
$$= -5 - 4\left(-\frac{81}{71}\right) + \frac{148}{71}$$
$$= -5 + \frac{472}{71} = \frac{117}{71}.$$

Hence, the solution is $\quad x = \frac{117}{71}, y = -\frac{81}{71}, z = \frac{148}{71}.$

Example 6. *Solve the following system by Gauss's elimination method*

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16.$$

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Solution. We have

$$2x + y + z = 10 \qquad ...(1)$$
$$3x + 2y + 3z = 18 \qquad ...(2)$$
$$x + 4y + 9z = 16. \qquad ...(3)$$

Divide (1) by 2 and subtract after and multiplied by 3 from (2) and subtract from (3), we get

$$x + \frac{1}{2}y + \frac{1}{2}z = 5 \qquad ...(4)$$

$$\frac{1}{2}y + \frac{3}{2}z = 3 \qquad ...(5)$$

$$\frac{7}{2}y + \frac{17}{2}z = 11. \qquad ...(6)$$

Now divide (5) by $\frac{1}{2}$ and then subtract it after multiplied by $\frac{7}{2}$ from (6), we get

$$x + \frac{1}{2}y + \frac{1}{2}z = 5 \qquad ...(7)$$

$$y + 3z = 6 \qquad ...(8)$$

$$-2z = -10. \qquad ...(9)$$

From back substitution from (9), (8) and (7), we get

$$z = 5$$

and

$$y + 3z = 6$$

$$y + 3(5) = 6$$

$$y = 6 - 15$$

$$y = -9$$

and

$$x + \frac{1}{2}(-9) + \frac{1}{2}(5) = 5$$

$$x = 5 + \frac{9}{2} - \frac{5}{2} = 7.$$

Hence, the solution is $x = 7, y = -9, z = 5$.

Example 7. *By Gauss's elimination method, solve*

$$4x + 11y - z = 33$$

$$x + y + 4z = 12$$

$$8x - 3y + 2z = 20.$$

Solution. Given equation are

$$x + y + 4z = 12 \qquad ...(1)$$

$$4x + 11y - z = 33 \qquad ...(2)$$

$$8x - 3y + 2z = 20. \qquad ...(3)$$

Eliminating x from (2) and (3), so subtract (1) after multiplied by 4 and 8 from (2) and (3), respectively, we get

$$x + y + 4z = 12 \qquad ...(4)$$

$$7y - 17z = -15 \qquad ...(5)$$

$$-11y - 30z = -76. \qquad ...(6)$$

Now divide (5) by 7 and then subtract if after multiplied by − 11 from (6), the equation (1) to (3) reduce to

$$x + y + 4z = 12 \qquad ...(7)$$

$$y - \frac{17}{7}z = -\frac{15}{7} \qquad ...(8)$$

$$-\frac{397}{7}z = -\frac{697}{7}. \qquad ...(9)$$

By back substitution from (9), (8) and (7), we get

$$z = \frac{697}{397}.$$

From (8)
$$y = -\frac{15}{7} + \frac{17}{7} z$$

$$y = -\frac{15}{7} + \frac{17}{7}\left(\frac{697}{397}\right) = \frac{1}{7}\left(\frac{5894}{397}\right)$$

$$y = \frac{5894}{2779}.$$

From (7)

$$x + y + 4z = 12$$

$$x + \frac{5894}{2779} + 4\left(\frac{697}{397}\right) = 12$$

$$x = 12 - \frac{5894}{2779} - 4\left(\frac{697}{397}\right) = \frac{7938}{2779}.$$

Hence, the solution is

$$x = \frac{7938}{2779} = 2{\cdot}856$$

$$y = \frac{5894}{2779} = 2{\cdot}121$$

$$z = \frac{697}{397} = 1{\cdot}756.$$

Example 8. *Solve by Gauss elimination method*

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13.$$

Solution. Given equation are

$$x + 2y + z = 3 \qquad \text{...(1)}$$
$$2x + 3y + 3z = 10 \qquad \text{...(2)}$$
$$3x - y + 2z = 13. \qquad \text{...(3)}$$

Here pivot element of (1) is 1 so no need for dividing (1). Now eliminating x from (2) and (3) by subtracting (1) after multiplied by 2 and 3 respectively from (2) and (3), we get

$$x + 2y + z = 3 \qquad \text{...(4)}$$
$$-y + z = 4 \qquad \text{...(5)}$$
$$-7y - z = 4. \qquad \text{..(6)}$$

Now eliminating y from (6) with the help of (5) by subtracting (5) after multiplied by – 7 from (6), the equations (1) to (3), reduce to

$$x + 2y + z = 3 \qquad \text{...(7)}$$
$$-y + z = 4 \qquad \text{...(8)}$$
$$6z = 32. \qquad \text{...(9)}$$

By back substitution from (9), (8) and (7), we get

From (9)
$$z = \frac{32}{6}$$

From (8)
$$-y = 4 - z$$

$$= 4 - \frac{32}{6} = -\frac{8}{6}.$$

$$y = \frac{8}{6}.$$

$\therefore$

$$x + 2y + z = 3$$

From (7)

$$x + 2\left(\frac{8}{6}\right) + \frac{32}{6} = 3$$

$$x = 3 - \frac{32}{6} - \frac{16}{6}$$

$$= \frac{18 - 48}{6} = -\frac{30}{6}$$

$$x = -5.$$

Hence, the solution is

$$x = -5, y = \frac{8}{6}, z = \frac{32}{6}$$

$$x = -5, y = \frac{4}{3}, z = \frac{16}{3}.$$